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INTRODUCTION TO DIFFERENTIAL EQUATIONS WITH MOVING SINGULARITIES

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Consider the differential system

$$\text{(1)}\qquad \begin{cases} y' = F(t, \epsilon, y, x) \\ \epsilon^{h}\phi(t, \epsilon)x' = G(t, \epsilon, y, x) \end{cases}$$

with the following hypothesis that we will call H.

H — (i) y is an m dimensional column vector and x is an n dimensional column vector. $F(t, \epsilon, y, x)$, $G(t, \epsilon, y, x)$ are continuously differentiable vector functions in the domain D, where

$$D = \{0 \leqq t \leqq 1, 0 \leqq \epsilon \leqq \epsilon_0, (\epsilon_0 > 0), (\|y\| + \|x\|) < \infty\},$$

(ii) $h \geqq 0$ (h need not be an integer) and $\phi(t, \epsilon)$ is a continuously differentiable scalar function in D,

(iii) $\phi(t, \epsilon) > 0$ for $0 \leqq t \leqq 1$, $0 < \epsilon < \epsilon_0$, $\phi(t, 0) \cdot \phi(0, \epsilon) \not\equiv 0$. If $h = 0$ we demand that $\phi(0, 0) = 0$ and

$$\lim_{\epsilon \to 0^+} \int_0^t \frac{d\eta}{\phi(\eta, \epsilon)} = +\infty \text{ uniformly}$$

for

$$0 < \delta \leqq t \leqq 1.$$

In the case when $h > 0$ and $\phi(t, \epsilon) \equiv 1$, we recognize the singular perturbation systems, in case $h = 0$ and $\phi(t, \epsilon) = (t + \epsilon)^m$, we have by letting $\epsilon = 0$, the familiar singular systems of mathematical physics. We call (1) a differential equation with *moving singularities* since the location of the zeros of $\phi(t, \epsilon)$ may depend on ϵ. For example,

$$\text{(2)}\qquad \begin{aligned} y_1' &= y_2 \\ \epsilon(t^2 + \epsilon)y_2' &= +y_1^3 - y_2, \end{aligned}$$

$$\text{(3)}\qquad \begin{aligned} y_1' &= y_2 \\ (t + \epsilon)^2 y_2' &= y_1 + y_2, \end{aligned}$$

$$\text{(4)}\qquad \begin{aligned} y_1' &= y_1 + 2y_2 \\ \epsilon y_2' &= 4y_1 + (t^2 + \epsilon)y_2 \end{aligned}$$

are special types of differential equations with moving singularities.