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ω-SEMIGROUPS

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§1. **Introduction.** Let ϵ stand for the set of non-negative integers (*numbers*), V for the class of all subcollections of ϵ (*sets*), Λ for the set of isols, and Ω for the class of all recursive equivalence types (RET). The relation of inclusion is denoted $\subset$, α recursively equivalent to β by $\alpha \simeq \beta$, for sets α and β, and the RET of α by Req (α). For the purpose of this paper we say a *semigroup* is an ordered pair (α, p), where (i) $\alpha \subset \epsilon$ and (ii) p is a semigroup operation (i.e., an associative binary multiplication) on $\alpha \times \alpha$. An *ω-semigroup* is a semigroup (α, p), where p can be extended to a partial recursive function of two variables. The concept of an ω-semigroup is a recursive analogue of a semigroup and is a generalization of an ω-group. In this paper, the author shows (T2), that there are ω-semigroups which are groups but not ω-groups; but that all periodic ω-semigroups which are groups, are ω-groups (T1). Theorems T3, T5, T6, T7, and T11 give conditions for an ω-semigroup to be an ω-group. The recursive analogues of regular semigroup, inverse semigroup, and right group [ω-regular ω-semigroup, inverse ω-semigroup, and ω-right group] are studied in sections §5, §6, §8 respectively, with particular attention paid to $T(\alpha)$, the analogue of the regular semigroup of all mappings from α into α, and $I(\alpha)$, the analogue of the symmetric inverse semigroup on α. Theorems T17 and T22 relate ω-regular ω-semigroups to ω-groups and T28 relates ω-regular ω-semigroups to inverse ω-semigroups. In T42 and T43 we have two nice characterizations of an ω-right group and T45 shows that a periodic ω-semigroup that is a right group is an ω-right group. Finally section §7 gives a brief introduction to the ω-homomorphism theory of ω-semigroups. The author wishes to thank the referee for his helpful suggestions.

§2. **Basic concepts and notations.** The reader of this paper is assumed to be familiar with the notation and basic results of [2], [6], and [7].

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