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T-FRACTIONS FROM A DIFFERENT POINT OF VIEW

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The continued fractions of the form

$$(1)\qquad 1 + d_0 z + \frac{z}{1 + d_1 z} + \cdots + \frac{z}{1 + d_n z} + \cdots$$

were introduced by W. J. Thron in 1948 [5], and are referred to as T-fractions.

Between the class of T-fractions and the class of formal power series $1 + c_1 z + c_2 z^2 + \cdots$ there is a one-to-one correspondence. The formal approximation is however slow, and none of the approximants are in the Padé table. But on the other hand, the T-fractions have several interesting properties, such as a Stieltjes integral representation (under certain conditions on the parameters d_n) and simple convergence properties associated with properties of the sequence (d_n) [2], [3], [5].

The purpose of this paper is to present some convergence theorems of a different type, namely theorems for the T-fraction expansions of certain functions.

Let f_0 be holomorphic in some neighborhood D_0 of the origin and normalized by $f_0(0) = 1$. Let furthermore (f_n) be the sequence of functions defined by

$$(2)\qquad f_n(z) = 1 + (f_n{}'(0) - 1)z + \frac{z}{f_{n+1}(z)},\ z \neq 0,\ f_{n+1}(0) = 1.$$

Then every f_n is analytic in some neighborhood D_n of the origin, and with $d_n = f_n{}'(0) - 1$, the identity

$$(3)\quad f_0(z) = 1 + d_0 z + \frac{z}{1 + d_1 z} + \cdots + \frac{z}{1 + d_{n-1} z} + \frac{z}{f_n(z)}$$

holds in a neighborhood of the origin, and the continued fraction (1) is the T-fraction expansion of f_0.

The theorems to be presented are of the form:

Boundedness property of $f_0 \Rightarrow$ convergence property of the T-fraction expansion of f_0. The first result of this type was proved in 1964 [6]:

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