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SOME APPLICATIONS OF PADÉ APPROXIMANTS TO QUANTUM FIELD THEORY MODELS*

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1. **Introduction.** We recall that a power series

$$P(z) = c_0 + c_1 z \cdots \tag{1}$$

is a series of Stieltjes iff $D(0, n)$ and $D(1, n) > 0$ [10] where

$$D(m, n) = \det \begin{bmatrix} c_m & \cdots & c_{m+n} \\ \vdots & \ddots & \\ c_{m+n} & & c_{m+2n} \end{bmatrix} . \tag{2}$$

Let $P(z)$ be a series of Stieltjes, then it may be represented as

$$g(z) = \int_{-\infty}^{\infty} \frac{d\sigma(x)}{1 - zx} \tag{3}$$

with $d\sigma(x) \geqq 0$ and $c_n = \int_{-\infty}^{\infty} x^n \, d\sigma(x)$. The problem of constructing $\sigma(x)$ from a knowledge of the c_n is the Hamburger moment problem while the requirement $d\sigma(x) = 0$ for $x < 0$ defines the Stieltjes moment problem. The moment problems are either determinate ($\sigma(x)$ unique) or indeterminate (infinitely many $\sigma(x)$'s). Note that the determinateness of the Hamburger moment problem implies the determinateness of the Stieltjes moment problem but that the converse is not true.

The $[n/n + j](z)$ Padé approximant to $P(z)$ is $P_n(z)/Q_{n+j}(z)$ where $P_n(z)$ is a polynomial of degree n and $Q_{n+j}(z)$ is a polynomial of degree $n + j$ in z. If the Stieltjes moment problem is determinate then $[n/n + j](z)$ converges as $n \to \infty$ to the unique function $g(z)$ for z in the cut plane $z \notin [0, \infty)$ and gives one an approximate method of constructing $\sigma(x)$ via the Stieltjes inversion formula which expresses $\sigma(x)$ in terms of $g(z)$.

In Sec. 2 we review the close connection between Padé approximants applied to series of Stieltjes and the theory of positive symmetric operators in a Hilbert space. This will allow us to (1) establish useful

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