ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 4, Number 2, Spring 1974

ON COMPUTATIONAL APPLICATIONS OF THE THEORY OF MOMENT PROBLEMS

SVEN-ÅKE GUSTAFSON*

SUMMARY. Many computational problems can be formulated as the task to evaluate a linear functional L for a given function φ when L is subject to a finite number of constraints.

In this paper we discuss tasks of this form. $L(\varphi)$ can be evaluated numerically either by approximating φ with linear combinations of a given system of functions $u_1, u_2, \cdots, u_n$ or by approximating L with a finite sum. In this way one can treat effectively such problems as the evaluation of a class of slowly convergent Fourier integrals, finding the limit value of sequences and the approximation of functions.

In our theoretical analysis we shall use the theory of the moment problem and consider generalizations of an optimization problem first studied by A. A. Markov and P. L. Čebyšev. We extend the results in various directions using the theory of semi-infinite programming.

1. **Introduction.** Let $[a, b]$ be a closed bounded interval and denote with $C[a, b]$ the space of functions f which are continuous on $[a, b]$ and normed by $\|f\| = \max_{a \leqq t \leqq b} |f(t)|$. Let L be a bounded linear functional defined on $C[a, b]$ and let $u_1, u_2, \cdots$ be a sequence of functions in $C[a, b]$. In this paper we shall discuss general and effective ways of solving:

TASK S: *Compute $L(\varphi)$ when $L(u_r) = \mu_r$, $r = 1, 2, \cdots$. The sequence $\mu_1, \mu_2, \cdots$ is given numerically and $\varphi(t)$ can be evaluated at any point t in $[a, b]$.* (No explicit representation of L is assumed to be known.)

THEOREM 1. *Let φ be a given function in $C[a, b]$ and $u_1, u_2, \cdots$ a sequence in $C[a, b]$ such that to every $\epsilon > 0$ one can find a finite linear combination $\sum_{r=1}^{N} c_r u_r$ meeting the condition*

$$(1) \qquad \left\| \sum_{r=1}^{N} c_r u_r - \varphi \right\| < \epsilon.$$

Let L be a bounded linear functional on $C[a, b]$ and put $\mu_r = L(u_r)$, $r = 1, 2, \cdots$. Then the value of $L(\varphi)$ is uniquely defined by the conditions

Received by the editors February 8, 1973.
*This research was financially supported by NSF under Grant GK-31833 and the Swedish Institute of Applied Mathematics, Stockholm.