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CONTINUATION OF FUNCTIONS BEYOND NATURAL BOUNDARIES*

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One encounters series of the form

$$f(z) = \sum_n A_n/(z - z_n) \tag{1}$$

in many researches concerning analytic continuation. A family of functions defined by some restriction on the A_n, such as

$$\sum \frac{\log \log(1/|A_n|)}{\log(1/|A_n|)} \text{ converges,} \tag{2}$$

is quasi-analytic provided that two functions belonging to the class and coinciding on an arc of curve on which both series (1) converge uniformly coincide everywhere.

Carleman [1] has shown that the class defined by (2) is quasi-analytic. Carleman says that Denjoy has shown by example that the class defined by

$$|A_\nu| < \exp(-\nu^{1/2-\epsilon}), \tag{3}$$

is not quasi-analytic. Since (2) is satisfied by

$$|A_\nu| < \exp(-\nu^{1+\epsilon}), \tag{4}$$

there exists a gap in our knowledge: for example, is the class defined by

$$|A_\nu| < \exp(-\nu^{1/2}) \tag{5}$$

quasi-analytic or not? I do not know whether or not this question has been answered since Carleman wrote his book in 1926.

I was led by these facts to study the convergence of the $[N/N + 1]$ Padé approximants to

$$f(z) = \sum_{n=2}^{\infty} \sum_{m=1}^{n-1} e^{-n} \Big/ \left(z - \exp(2\pi i \frac{m}{n}) \right), \tag{6}$$

where m and n are relatively prime. I also studied the example

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