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CERTAIN INVARIANCE AND CONVERGENCE PROPERTIES OF THE PADÉ APPROXIMANT*

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We define the Padé approximant, invented by Jacobi [12], to the formal power series $f(x)$, by the equations

$$[L/M] = \frac{P_L(x)}{Q_M(x)} \tag{1}$$

$$Q_M(x)f(x) - P_L(x) = O(x^{M+L+1}) \tag{2}$$

$$Q_M(0) = 1 \tag{3}$$

where P_L and Q_M are polynomials of degree L and M respectively. This definition differs from the classical one of Frobenius [9] and Padé [14], in the use of (3). Under our definition the Padé approximants do not always exist, but an infinite number on each row, column, and diagonal of the Padé table always do exist [4].

The diagonal, $L = M$, Padé approximants satisfy the following invariance theorem,

THEOREM (INVARIANCE). *If $P_M(x)/Q_M(x)$ is the $[M/M]$ Padé approximant to $f(x)$, and $C + Df(0) \neq 0$, then*

$$\frac{A + B\left[P_M\left(\frac{\alpha y}{1+\beta y}\right) \Big/ Q_M\left(\frac{\alpha y}{1+\beta y}\right)\right]}{C + D\left[P_M\left(\frac{\alpha y}{1+\beta y}\right) \Big/ Q_M\left(\frac{\alpha y}{1+\beta y}\right)\right]} \tag{4}$$

is the $[M/M]$ Padé approximant to

$$\{A + Bf[\alpha y/(1+\beta y)]\}/\{C + Df[\alpha y/(1+\beta y)]\}. \tag{5}$$

The proof of this theorem is easily constructed by multiplying numerator and denominator by $(1+\beta y)^M\, Q_M(\alpha y/(1+\beta y))$. This operation reduces form (4) to the ratio of two polynomials of degree M, and thus the invariance theorem can be made to follow from the uniqueness theorem:

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