ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 4, Number 1, Winter 1974

THE NIELSEN FIXED POINT THEORY FOR NONCOMPACT SPACES

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Introduction. The Nielsen fixed point theory provides a lower bound for the number of fixed points of a map $f: X \to X$ on a compact metric ANR (absolute neighbourhood retract) ([5], [11], [15]). This lower bound, denoted by $N(f)$ (the Nielsen number of f) is a non-negative integer and is known to be homotopy invariant ([11], [15]). Recently, Brown [7], has shown that one can obtain a Nielsen theory on noncompact ANR's by requiring only that the maps and homotopies be compact (i.e., their images have compact closure).

The purpose of this paper is to establish a Nielsen theory for very general classes of self maps (§1) and to give a proof of the homotopy invariance of the Nielsen number. The motivation for this type of generality becomes clear in §3 where several interesting and analytically important examples of such classes are produced.

§1. **Preliminaries.** Given a self map $f: X \to X$, let $\Phi(f)$ represent the set of fixed points of f. We let I denote the closed unit interval. For a given homotopy $H: X \times I \to X$, we shall make use of the following maps: $\mathbf{H}: X \times I \to X \times I$ given by $\mathbf{H}(x, t) = (H(x, t), t)$; for $t \in I, h_1: X \to X$ defined by $h_t(x) = H(x, t)$; finally, for $r, s \in I$, $H^{r,s}: X \times I \to X$ defined by $H^{r,s}(x, t) = H(x, (1 - t)r + ts)$.

Let $\mathcal{G}$ be a class of self maps. An $\mathcal{G}$-homotopy is a map $H: X \times I \to X$ such that $H^{r,s} \in \mathcal{G}$ and $h_t \in \mathcal{G}$ for all $r, s, t \in I$. If H is an $\mathcal{G}$-homotopy, we say that h_0 and h_1 are $\mathcal{G}$-homotopic.

For a given class of self maps $\mathcal{G}$, we let $\mathcal{C}_{\mathcal{G}}$ be the class of all triples (X, f, U) where $f: X \to X$ is a map in $\mathcal{G}$, and U is an open subset of X which has no fixed points of f on its boundary.

Let $f: X \to X$ be a map and $H_* = \{H_p, \partial_p\}$ a rational homology theory defined for a category of spaces including X. Let $f_{*,p}: H_p(X) \to H_p(X)$ denote the induced homomorphism. We say that f has a

Received by the editors February 2, 1972.
AMS 1970 *subject classifications.* Primary 54H25; Secondary 47H10, 55C20.
It is a pleasure to acknowledge the encouragement given me by Professor Robert F. Brown during this writing. His advice and criticism have been invaluable.
[1]This work was supported by NSF Grant GU 1534.