## SOME C\*-ALGEBRAS WITH OUTER DERIVATIONS GEORGE A. ELLIOTT

1. In [9], Sakai has given an example of a simple C\*-algebra without unit every derivation of which is inner. Theorem 2 below shows that such a C\*-algebra cannot be separable.

Theorem 3, the main result of this paper, gives a complete description of separable liminary  $C^*$ -algebras every derivation of which is inner.<sup>1</sup>

A consequence of Theorem 3 is that a separable liminary  $C^*$ -algebra every derivation of which is inner is the direct sum of a commutative algebra and an algebra with unit. Theorem 2 shows that this implication holds for a separable primitive  $C^*$ -algebra, and a modification of the proof of Theorem 3 (see 4.3) shows that it holds for a separable  $C^*$ -algebra whose primitive spectrum is separated.<sup>2</sup>

Another consequence of Theorem 3 is that if every derivation of a separable liminary  $C^*$ -algebra is inner then each quotient of this C\*-algebra has this property.

2. THEOREM. Let A be a separable C\*-algebra. If A has a primitive quotient without unit then A has outer derivations.

**PROOF.** By [1] A has a commutative approximate unit, contained, say, in a commutative sub- $C^*$ -algebra B of A. Let t be a primitive ideal of A such that A/t does not have a unit. Then because B is separable and (B + t)/t does not have a unit, there is a bounded sequence  $(x_n)$  of elements of B whose images in (B + t)/t have norm one, and whose supports in the spectrum of B are compact, mutually disjoint, and, except for finitely many, disjoint from each fixed compact.

Claim. The inner derivation of A defined by  $\sum_{n=1}^{k} x_{2n}$ , k = 1, 2,  $\cdots$ , converges simply to an outer derivation.

To show convergence on all of A it is enough to show convergence on a dense subset of A. The set of  $x \in A$  such that yx = xy = x for some  $y \in B$  of compact support in the spectrum of B is dense in A, and for each such x,  $x_n x = x x_n = 0$  for all but finitely many n.

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<sup>&</sup>lt;sup>1</sup>Added in proof: It has been shown by Akemann, Elliott, Pedersen and Tomiyama (Derivations and multipliers of C\*-algebras, preprint) that in this theorem "liminary" can be replaced by "postliminary".

<sup>&</sup>lt;sup>2</sup>Added in proof: This implication has now been established for an arbitrary separable C\* -algebra (op. cit.).