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# STABILITY AND BIFURCATION IN FLUID DYNAMICS

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1. **Introduction.** During the last decade some of the instability phenomena in fluid dynamics discovered in the famous early experiments of Bénard [3] and Taylor [**59**] have found a mathematical interpretation in the proof that the stationary Navier-Stokes boundary-value problem is not in general uniquely solvable. Even for moderate values of the underlying similarity parameter (Reynolds number, Rayleigh number), infinitely many solutions exist which bifurcate from some basic flow at definite values of this parameter. The mathematical interest in this subject was stimulated by the paper of Velte [**61**], who proved by topological arguments the existence of bifurcating solutions for a certain convection problem. Later, Iudovich obtained the most complete results yet known for the Bénard and the Taylor problem ([**21**]—[**24**]). Closely connected to bifurcation phenomena is the question of which, among the many solutions, is the one actually observed. This is generally believed to be a stability problem. The fundamental result relating spectral properties of the Stokes equations and the stability of solutions of the Navier-Stokes equations is due to Prodi [**45**].

Simultaneous with the above-mentioned research, considerable effort was spent on the development of analytical methods for the study of those problems. Stuart proposed an amplitude expansion by which many of the results known today were obtained before they were rigorously proved [**14**]. Only recently has his method found mathematical justification to some extent [**19**].

This survey covers exclusively the mathematical part of bifurcation and stability in fluid dynamics, although other results are mentioned for comparison and stimulation of research. Another branch is excluded as well, time periodic motions of Tollmien-Schlichting type, since no concrete models are known for the theoretical results published recently.

The conception of this survey is to display the common functional analytic structure of the Taylor and Bénard problem. Most of the bifurcation models studied in fluid dynamics have the same structure.

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