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INVARIANT MEANS ON SUBSEMIGROUPS OF LOCALLY COMPACT GROUPS

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1. **Introduction.** Throughout this paper G will denote a locally compact group and locally null subsets of G are defined with respect to a fixed left Haar measure λ of G.

Recently J. W. Jenkins [5] shows that if S is an open subsemigroup of G and G is left amenable, then S is left amenable if and only if S has finite intersection property for open right ideals. In this paper, we shall prove an analogue result for any nonlocally null Borel measurable subsemigroups S of G, generalising a result of Frey [2] (see also [8, Theorem 3.5]) for discrete left amenable groups.

2. **Preliminaries and some notations.** For any subset A of a topological space Y, $\bar{A}$ will denote the closure of A in Y and 1_A will be the characteristic one function on A. The class of Borel sets in Y is the smallest σ-algebra of sets containing all open subsets of Y.

Let S be a *topological semigroup*, i.e., S is a semigroup with a Hausdorff topology such that, for each $a \in S$, the two mappings from S into S defined by $s \to as$ and $s \to sa$ for all $s \in S$ are continuous. Let $MB(S)$ be the space of bounded Borel measurable real valued functions on S equipped with the sup norm topology. For each $a \in S$, define two operators, r_a and l_a, from $MB(S)$ into $MB(S)$ by $r_af(s) = f(sa)$ and $l_af(s) = f(as)$ for all $s \in S$, $f \in MB(S)$. Let X be a closed subspace of $MB(S)$ containing 1_S. An element ϕ in X^*, the conjugate space of X, is a *mean* if $\phi(1_S) = \|\phi\| = 1$. Furthermore, the restriction of any element in the convex hull of $\{p_s; s \in S\} \subseteq MB(S)^*$ to X is called a *finite mean* on X, where $p_s(f) = f(s)$. As known [1] the set of finite means on X is weak* dense in the set of means on X. If X is invariant under l_a for each $a \in S$, then a mean ϕ on X is a *left invariant mean* (LIM) if $\phi(l_af) = \phi(f)$ for all $a \in S$ $f \in X$. S is *left amenable* if $MB(S)$ has a LIM.

A bounded continuous real valued function f on S is *uniformly*

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