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ON MORE RESTRICTED PARTITIONS

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1. **Introduction.** Recently Gupta [2] studied the function $g(n, m, h, k)$ which enumerates the number of partitions of n into exactly k summands each less than or equal to m and in which the number of distinct summands is exactly h. Earlier Cheema and Haskell [1] studied $p(n, r, m, k)$, the number of partitions of n into r summands such that each summand is less than or equal to k and greater than or equal to m. In this article it is shown that the above results can be generalized to study $g(n, l, m, h, k)$ (and other related functions) which enumerates the number of partitions of n into exactly k summands each less than or equal to m and greater than or equal to l in which the number of distinct summands is exactly h.

2. $g(n, l, m, h, k)$ is the coefficient of $x^n z^k t^h$ in

$$\prod_{i=l}^{m}\left(1+\frac{x^i z}{1-x^i z}t\right) \quad \text{i.e., in} \quad \prod_{i=l}^{m}\left(\frac{1+x^i z(t-1)}{1-x^i z}\right)=\frac{f(t)}{f(0)} \tag{2.1}$$

where

$$\begin{aligned} f(t) &= \prod_{i=l}^{m}(1+x^i z(t-1)) \\ &= 1+\begin{bmatrix} m-l+1 \\ 1 \end{bmatrix} x^{l-1}z(t-1) \\ &\quad +\begin{bmatrix} m-l+1 \\ 2 \end{bmatrix} x^{2(l-1)}z^2(t-1)^2 \\ &\quad +\cdots+\begin{bmatrix} m-l+1 \\ m-l+1 \end{bmatrix} x^{(m-l+1)(l-1)}z^{m-l+1}(t-1)^{m-l+1}, \end{aligned} \tag{2.2}$$

where

$$\begin{bmatrix} m \\ i \end{bmatrix}=\frac{(1-x^m)(1-x^{m-1})\cdots(1-x^{m-i+1})}{(1-x)(1-x^2)\cdots(1-x^i)}x^{i(i+1)/2}. \tag{2.3}$$

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