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SOLUTION OF THE ALMOST COMPLEX SPHERES PROBLEM USING K-THEORY

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1. **Introduction.** Let $F(n)$ denote $SO(2n)/U(n)$. We shall abbreviate $K_C^*(X)$ to simply $K(X)$. Finally, $K(X; Q)$ represents $K(X) \otimes Q$, where Q is the field of rational numbers.

The two results of this paper are the following:

1.1. A description of $K(F(n); Q)$.

1.2. A new proof that the only almost complex spheres are S^2 and S^6.

The first proof that the only almost complex spheres are S^2 and S^6 was given by Borel and Serre in [5]; their proof used the Steenrod reduced power operations. Our proof uses 1.1 and the Chern character.

The contents of this paper are as follows: §2 contains background material. In §3 we calculate $K(F(n); Q)$. We also indicate a method for calculating $K(F(n))$. §4 is devoted to 1.2.

This material constitutes part of the author's doctoral thesis [2]. I wish to thank Professor Albert Lundell for his advice.

2. **Background.** A complete reference for this section is [8].

A $2n$-dimensional real manifold M is *almost complex* if its tangent sphere bundle

$$S^{2n-1} \to T(M) \to M$$

with structural group $O(2n)$ is equivalent in $O(2n)$ to a bundle with structural group $U(n)$. This happens if and only if the associated bundle with fibre $F(n)$ has a cross section.

For the sphere S^{2n}, the tangent sphere bundle is

$$S^{2n-1} \to SO(2n+1)/SO(2n-1) \to S^{2n}.$$

The associated principal bundle is

$$SO(2n) \to SO(2n+1) \to S^{2n},$$

and, since $SO(2n+1)/U(n) \approx F(n+1)$, the associated bundle with

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