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INDEPENDENT CLASSES OF SEMIPRIMAL ALGEBRAS

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1. **Introduction.** Recent investigations in the area of universal algebra have focused attention on the conditions under which an algebra subdirectly decomposes within a given class of algebras. See, for example, Astromoff [1], Foster and Pixley [2], [3], [5], Gould and Grätzer [6], Hu [8], and Jónsson [9]. In particular, it has been shown that each cluster K of primal or, more generally, semiprimal algebras determines a certain unique subdirect factorization for each algebra which satisfies the identities common to some finite subset of K. Primal clusters (see [3], [10]-[12], [14], and [15]) are known to exist in great profusion.

In this paper we show the existence of clusters of semiprimal algebras of rather diverse nature, thereby enhancing the scope of applicability of the more general semiprimal theory. Our main result is

THEOREM 1. *A family K of semiprimal algebras is a cluster if* (a) *each member of K is strongly surjective,* (b) *the members of K have pairwise nonisomorphic structures, and* (c) *for each $A \in K$, the intersection of all subalgebras of A is nonempty.*

We prove this result with an eye on the techniques of O'Keefe [10]-[12]. It is shown in §3 that K is a cluster if its members are pairwise independent. This pairwise independence for members of K is then established in §4.

2. **Basic definitions.** Let S be a fixed finitary species (or type) of universal algebra. All algebras to be considered are assumed to be of species S.

(1°) A *term* (or *expression*) is any indeterminate symbol x, y, x_1, y_1, $\cdots$ or any finite composition of indeterminate symbols via the primitive operations of S. All terms are written as italic caps, F, G, H, etc. We write $F = G(A)$ when two terms F and G agree as functions within an algebra A.

(2°) Any finitary mapping $f : A^n \to A$ for arbitrary n is called an *A-function.* Such a function is said to be *conservative* if for each subalgebra A' of A, $f(a_1, \cdots, a_n) \in A'$ whenever $a_1, \cdots, a_n \in A'$. An algebra A is *primal* (respectively, *semiprimal*) if it is finite, contains

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