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THE DOUBLE CENTRALIZER PROPERTY IS CATEGORICAL[1]

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The purpose of this note is to prove the result stated in the title. This answers a question raised by E. A. Walker at the summer symposium on ring theory at Appalachian State University in August, 1969, as to whether being QF-1 is categorical. This also seems timely since many papers have appeared recently studying the double centralizer property for modules, for instance [2], [4], and [6].

Let R and S be associative rings with identity such that the categories ${}_R\mathcal{M}$ and ${}_S\mathcal{M}$ of left R-modules and left S-modules respectively are equivalent. Then by [7, Theorem 3.5] or [1, Theorem 3.4, p. 62] there is a right R-progenerator P_R with $S \simeq \mathrm{End}_R(P_R)$ such that the functor $F = {}_SP_R \otimes_R (-) : {}_R\mathcal{M} \to {}_S\mathcal{M}$ gives the equivalence; we say R and S are Morita equivalent via ${}_SP_R$.

Let M be a left R-module and let $C = \mathrm{End}_R({}_RM)$. Then the map $C \to \mathrm{End}_S({}_SP \otimes_R M)$ via $f \to 1_P \otimes f$ is a unital ring isomorphism so we identify C with $\mathrm{End}_S(P \otimes_R M)$. Let $D = \mathrm{End}_C(M_C)$ and $E = \mathrm{End}_C(P \otimes_R M_C)$ be the double centralizers of ${}_RM$ and ${}_SP \otimes_R M = {}_SF(M)$ respectively; note that we write homomorphisms opposite scalars. We say that M has the *double centralizer property* (DCP) if the natural map η: $R \to \mathrm{End}_C(M_C)$ via $\eta(r)(m) = rm$ is onto. Equivalently, if $\mathrm{Ann}_R(M) = \{r \in R \mid rm = 0 \text{ for all } m \in M\}$, M has the DCP if the natural map $R/\mathrm{Ann}_R(M) \to \mathrm{End}_C(M)$ is an isomorphism.

Finally, we say that two modules N and N' are *similar* if each is isomorphic to a direct summand of a finite direct sum of copies of the other and in this case we write $N \sim N'$.

THEOREM. *Let R and S be Morita equivalent via the module ${}_SP_R$. If a left R-module ${}_RM$ has the double centralizer property, so does ${}_SF(M) = {}_SP \otimes_R M$.*

PROOF. Since P_R is a progenerator, $P_R \sim R_R$, so $P \otimes_R M \sim R \otimes_R M \cong M$ as abelian groups, and the action of C on $P \otimes_R M$ makes

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[1]K. R. Fuller tells us this result also appears in an unpublished paper by Morita and Tachikawa. We feel our proof is worth recording since it was obtained independently and no proof is available in print.