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TOPICS ON HOLOMORPHIC CORRESPONDENCES[1]

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Correspondences, i.e. set valued mappings, occur in a natural way in the field of complex function theory. First of all multivalued functions arise from single valued functions via analytic continuation. Another phenomenon is that meromorphic functions of more than one variable can have points of indeterminacy. As an example, consider the meromorphic function z_1/z_2 on the space C^2 of two complex variables z_1, z_2. Outside the origin of C^2 this defines a mapping into the extended complex plane $\bar{C}$. But for each value in $\bar{C}$ there is a complex line through the origin on which z_1/z_2 is constantly equal to that value. Hence it is natural to assign $\bar{C}$ as the "value" of z_1/z_2 at the origin. In this way we obtain a correspondence which even has the additional property that its graph is an analytic set in $C^2 \times \bar{C}$.

This leads to the notion of holomorphic correspondences. The purpose of the present note is to deal with this concept from several points of view.

First we discuss some definitions of continuity and other topological aspects. Next we consider holomorphic correspondences and their extensions over analytic exceptional sets. Another method of extending holomorphic correspondences is to apply a reflection principle. This is described in the last section. We sketch a new proof of a theorem first shown by Tornehave and discuss applications and generalizations.

1. **Topological aspects.**

1.1. Let $X, Y, \cdots$ be sets. A *correspondence* from X to Y is a triple $f = (X, G, Y)$ where G, called the *graph* of f and denoted also G_f, is a subset of $X \times Y$. $f = (X, G, Y)$ assigns to any $x \in X$ the subset $f(x) := \{y \in Y : (x, y) \in G\}$ of Y; conversely, if to every $x \in X$ a subset M_x of Y is assigned, then there is a unique correspondence f from X to Y such that $M_x = f(x)$, one has $G_f = \{(x, y) \in X \times Y : y \in M_x\}$. For $A \subset X$ we set $f(A) := \bigcup_{x \in A} f(x)$. f is called empty if $G_f = \emptyset$ or, equivalently, if $f(X) = \emptyset$.

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