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WEAK PROBABILITY DISTRIBUTIONS ON REPRODUCING KERNEL HILBERT SPACES

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1. **Introduction.** In the experimental sciences, the problem of estimating a function from an approximate knowledge of a finite number of observations occurs very frequently. If the observational values were known exactly the techniques of Optimal Approximation [1] could be employed directly, but usually the given information is subject to experimental error, which may often be regarded as probabilistic in nature. One is thus led to consider a space of all the functions which might conceivably have given rise to the observations, and the possibility of constructing a probability measure on this space.

In choosing a curve of unknown algebraic form to fit experimental data there is a widespread predilection favoring "smoother" curves over those which are "not so smooth", presumably expressing the intuitive feeling that, if any given set of data could have arisen from either a smooth curve or a rough curve, then the former is "more likely" than the latter. Also in choosing such curves, one would feel that curves of equal norm should be equally likely. The first of these two considerations suggests a probability measure that favors smooth functions over unsmooth ones by assigning high probability to curves with small norms and low probability to curves with large norms. It would seem, then, that the choice of a Gaussian measure in which the functional $\exp\{-\|h\|^2\}$ plays the role of the relative likelihood of a member h of some Hilbert space is likely to be useful in numerical practice.

The second consideration carries us even further toward a Gaussian measure. To ask that our measure make curves of equal norm equally likely is to ask that it be invariant under unitary transformations.

It has been shown in [13] that in the setting we are about to consider a unitarily invariant, μ, must be of the form $\mu(\,\cdot\,) = \int_0^\infty \mu_\lambda(\,\cdot\,)\alpha(d\lambda)$ where $\{\mu_\lambda\}$ is a family of canonical Gaussian measures and α is a probability measure on $[0, \infty)$.

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