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LOCAL ALGEBRAIC INVARIANTS OF Δ-SETS

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In this note we define homology and homotopy groups at a vertex of a Δ-set. The theory is developed parallel to S. T. Hu's theory [1] of homology and homotopy groups at a point of a topological space. If one considers the space S of continuous functions of one differentiable manifold into another, it is well known that S is a differentiable manifold (usually modelled on some infinite dimensional linear space). When one attempts to put a piecewise linear structure on some piecewise linear function space, the attempt fails since the induced topology is wrong. Sacrificing the geometry, one considers these spaces as Δ-sets and the homotopy structure remains. The natural question arises to what extent can these Δ-sets be thought of as manifolds. The answer obtained in this paper for Δ-sets which naturally arise in piecewise linear topology is that the local homotopy and homology theory in the sense of Hu is similar.

In the last five sections, we calculate local invariants of some well-known Δ-sets.

1. **Δ-sets.** We recall some definitions and results from the Δ-set theory of C. P. Rourke and B. J. Sanderson [5] (or equivalently, the quasisimplicial theory of C. Morlet [4]). Note that, except in §8, we could have used semisimplicial sets; however the Δ-sets are easier to handle.

Let Δ^n denote the standard n-simplex with ordered vertices $v_0, v_1, \cdots, v_n$. The ith *face map* $\partial_i : \Delta^{n-1} \to \Delta^n$ is the order-preserving simplicial embedding which omits v_i. Δ is the category whose objects are Δ^n, $n = 0, 1, \cdots$, and whose morphisms are generated by the face maps. A *Δ-set* (*Δ-group*) is a contravariant functor from Δ to the category of sets (groups). A *Δ-map* between Δ-sets is a natural transformation between the functors.

If X is a Δ-set, $X^n = X(\Delta^n)$ is the set of n-simplexes and the maps $\partial_i \equiv X(\partial_i)$ are called *face maps*. A 0-simplex will also be called a *vertex*. We shall be interested in pointed Δ-sets $(X, *)$ in which we distinguish a simplex $*^n \in X^n$ for each n and designate $* \subset X$, the

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