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EVERYWHERE WILD CELLS AND SPHERES

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1. **Introduction.** Wild cells of all possible codimensions in E^n, $n \geqq 3$, were first constructed by W. A. Blankinship in [7]. Morton Brown [9] made a nice application of a result of J. J. Andrews and M. L. Curtis [3] to construct wild cells and spheres of all possible codimensions in E^n, $n \geqq 3$. Although Brown did not go into detail on his suggested method for producing wild codimension two spheres, such spheres had previously been given by J. C. Cantrell and C. H. Edwards in [11]. The work of Cantrell and Edwards was carried further by Ralph Tindell [19] to obtain some interesting wild embeddings in codimension two. Everywhere wild arcs in E^3 have been constructed by R. H. Bing [6] and W. R. Alford [1]. Also, everywhere wild arcs have been constructed in E^n, $n \geqq 4$, [9].

In this paper we establish the following result.

THEOREM. *In E^n, $n \geqq 3$, there are cellular, everywhere wild cells and everywhere wild spheres of all codimensions.*

Thus, it is immedate that there are also closed, everywhere wild strings and half-strings of all codimensions. In §6 we obtain noncellular everywhere wild cells. Finally, in §7, everywhere wild arcs are exhibited in E^n which pierce a locally flat $(n-1)$-sphere at a single point. The author expresses gratitude to C. E. Burgess and T. M. Price for helpful suggestions regarding this paper. For applications of this paper see [12], [17] and [18].

2. **Definitions.** Euclidean n-space is denoted by E^n, $E_+{}^n = E^{n-1} \times [0, \infty) \subset E^n$, $I^n = \{(x_1, x_2, \cdots, x_n) \mid -1 \leqq x_i \leqq 1, \quad i = 1, 2, \cdots, n\} \subset E^n$, and S^n is the boundary of I^{n+1}. An *n-string, n-half-string, n-cell, n-sphere* is a set which is homeomorphic to E^n, $E_+{}^n$, I^n, S^n, respectively. A topological k-manifold M in E^n is said to be *locally flat* at a point $x \in M$ if there is a neighborhood U of x in E^n such that the pair $(U, U \cap M)$ is homeomorphic to (E^n, E^k) if $x \in \operatorname{Int} M$ or to $(E^n, E_+{}^k)$ if $x \in \operatorname{Bd} M$. M is said to be *locally tame* at $x \in M$ if there is a neighborhood U of x in E^n and a homeomorphism of U onto E^n which carries $U \cap M$ onto a subpolyhedron of E^n. M

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