ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 2, Number 2, Spring 1972

INVERSES, LOGARITHMS, AND IDEMPOTENTS IN $M(G)$[1]

JOSEPH L. TAYLOR[2]

Let $M(R)$ denote the measure algebra on the line considered as a Banach algebra under convolution. In [**16**] we proved that if $\mu \in M(R)$ and μ is invertible, then μ has a factorization $\mu = \eta^k * \delta_x * \exp(\omega)$, where $k \in Z$, $x \in R$, $\omega \in M(R)$, and η can be chosen to be any measure in $L^1(R) + C\delta_0$ whose Fourier transform is non-vanishing and has winding number one about zero. This result implies that the group $M(R)^{-1}/\exp(M(R))$ is isomorphic to $Z \oplus R$. Since the numbers k and x in the above factorization can be explicitly determined from μ, this result completely characterizes the invertible measures in $M(R)$ which have logarithms in $M(R)$.

The above result is a special case of a general factorization theorem proved in [**16**] for any commutative convolution measure algebra — in particular, for all algebras $M(G)$ for G a locally compact abelian (l.c.a.) group or $M(S)$ for S a locally compact abelian topological semigroup. This theorem is proved using the Arens-Royden theorem [**1**], [**8**], and a result in [**16**] which characterizes the cohomology groups of the maximal ideal space of any measure algebra. Another consequence of this result is a new proof of Cohen's idempotent theorem [**3**].

In [**17**] using some of the same techniques we proved that if a measure $\mu \in M(G)$ is invertible in $M(G)$ then its inverse must lie in a certain "small" subalgebra of $M(G)$ containing μ. This greatly simplifies the problem of determining the spectrum of an element of $M(G)$.

Unfortunately, the above results rely heavily on the specialized machinery developed in [**11**], [**12**], [**13**], and [**14**] for the study of convolution measure algebras. Also, the proof of the factorization theorem in [**16**] uses a considerable amount of sheaf theory and algebraic topology. Thus, the student of harmonic analysis who wishes to understand these results is faced with a discouraging amount of machinery to wade through.

Received by the editors October 13, 1970.

AMS 1970 *subject classifications.* Primary 43A10, 43A05.

[1]Research partially supported by the United States Air Force, Air Force Office of Scientific Research, under AF-AFOSR 1313-67.

[2]The author currently holds an Alfred P. Sloan Foundation fellowship.