ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 2, Number 1, Winter 1972

PRINCIPAL SUBMATRICES. VIII. PRINCIPAL SECTIONS OF A PAIR OF FORMS[1]

R. C. THOMPSON

ABSTRACT. Let A, C be n-square Hermitian matrices, with C positive definite. Let A_i, C_i denote the principal submatrices obtained by deleting row and column i. In this paper new links are obtained between the roots of the determinantal equations $\det(\lambda C - A) = 0$, $\det(\lambda C_i - A_i) = 0$, $i = 1, \cdots, n$.

Let A be an n-square Hermitian matrix. Let $A(i \mid i)$ denote the principal submatrix of A obtained by deleting from A both row i and column i. In certain earlier papers in this series, links between the roots of

$$(1) \qquad \det(\lambda I_n - A) = 0$$

(the eigenvalues of A) and the roots of

$$(2) \qquad \det(\lambda I_{n-1} - A(i \mid i)) = 0, \qquad i = 1, 2, \cdots, n,$$

(the eigenvalues of $A(i \mid i)$) have been studied. It is of course true that, for each fixed i, the roots of (2) interlace the roots of (1). This well-known fact goes back to Cauchy, and for this reason these interlacing inequalities are often called the Cauchy inequalities.

Let C be an n-square positive definite Hermitian matrix. In this paper we study the roots of the equation

$$(3) \qquad \det(\lambda C - A) = 0$$

and their links to the roots of all of the equations

$$(4) \qquad \det(\lambda C(i \mid i) - A(i \mid i)) = 0, \qquad i = 1, 2, \cdots, n.$$

The equation (3) arises when one attempts a simultaneous diagonalization of a pair of quadratic forms having coefficient matrices C and A. (The possibility of this simultaneous diagonalization is important in applied mathematics, especially in mechanics — see [11].) Suppressing the same variable in each of these forms, the correspond-

Received by the editors July 20, 1970.
AMS 1970 *subject classifications*. Primary 15A39, 15A42, 15A63.
Key words and phrases. Linear algebra, principal submatrix, eigenvalue inequalities.
[1]The preparation of this paper was supported in part by the U.S. Air Force Office of Scientific Research, under Grant 698-67.