ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 2, Number 1, Winter 1972

PERTURBATION AND APPROXIMATION THEORY FOR HIGHER-ORDER ABSTRACT CAUCHY PROBLEMS

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1. **Introduction and summary.** Given a (possibly unbounded) linear operator A on a Banach space B, and a polynomial $P(\lambda, s) = \sum_0^{i=m;j=l} c_{ij}\lambda^i s^j$, there is defined the abstract Cauchy problem

$$(0.1)\qquad \begin{aligned} P(d/dt, A)u &= 0 && \text{for } t > 0, \\ (d/dt)^j \big|_{t=0} u &= f_j && \text{for } 0 \leqq j \leqq m-1. \end{aligned}$$

It has been shown in [D] that if A generates a group $T(t)$ and if

$$(0.2)\qquad \int_{-\infty}^{\infty} \sum_{k=0}^{m-1} T(s) f_k \hat{g}_k(t, s)\, ds$$

converges, where $\hat{g}_k(t, s)$ is the solution of (0.1) in the special concrete case $A = -d/dx$, $f_j = \delta_{jk}\delta(x)$, then u is given by (0.2). If $\hat{g}_k$ is a generalized function, (0.2) is interpreted by integration by parts.

A number of concrete Cauchy problems may conveniently be studied in terms of (0.1). This was done in [M] for one-dimensional parabolic equations of arbitrary order whose coefficients are measurable functions of x; other applications are mentioned below.

In the present work we exploit formula (0.2) to study two types of perturbation problems for (0.1). In the last section, we replace the fixed generator A by an approximate generator A_ϵ. It is natural to suppose that if A_ϵ generates a group $T_\epsilon(t)$ and $T_\epsilon(t) \to T(t)$, then the corresponding solution u_ϵ converges to u. This is often true, but for some P we will see that it requires an extra restriction on the data f.

Most of our work is concerned with perturbations of the polynomial P; i.e., we keep A fixed and let the coefficients c_{ij} depend on ϵ. In particular, we allow the leading coefficients to vanish as $\epsilon \to 0$, so that *singular* as well as regular perturbations are included in our theorems.

For technical reasons, we find it convenient to treat several cases, depending on the "type" of P_ϵ and of P; see Friedman [C] or Gel'fand-Shilov [B] for definitions and properties. The details vary, depending on whether the approximating or limiting polynomial is hyperbolic,

Received by the editors May 6, 1970 and, in revised form, July 10, 1970.
AMS 1970 *subject classifications.* Primary 34G05, 35B20, 35B25, 41A65.