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PROPERTIES OF PÓLYA PEAKS

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1. **Introduction.** In this note, $G(t)$ will represent a positive, nondecreasing, unbounded, continuous function defined for all real $t \geqq t_0 \geqq 0$.

The *order* λ and the *lower order* μ of the function $G(t)$ are defined by the relations

$$\limsup_{t\to\infty} \frac{\log G(t)}{\log t} = \lambda, \qquad \liminf_{t\to\infty} \frac{\log G(t)}{\log t} = \mu.$$

An increasing sequence

$$r_1, r_2, \cdots, r_m, \cdots$$

is said to be a *sequence of Pólya peaks of order* α $(0 \leqq \alpha < \infty)$ for the function $G(t)$ if it is possible to find sequences $\{r_m'\}$, $\{r_m''\}$ such that

$$r_m' \to \infty, \qquad r_m/r_m' \to \infty, \qquad r_m''/r_m \to \infty$$

and such that

$$(1.1) \qquad G(t)/t^\alpha \leqq (1 + o(1))G(r_m)/r_m^\alpha \quad (m \to \infty,\ r_m' \leqq t \leqq r_m'').$$

It is required that the error term $o(1)$ in (1.1) approaches zero uniformly as t tends to infinity in the intervals $[r_m', r_m'']$.

Pólya peaks have been used by Edrei and other authors [**6**], [**7**], [**10**] to establish interesting results on the value distribution theory of meromorphic functions in the plane; in particular, inequality (1.1) is useful in the estimation of integral transforms occurring naturally in this theory.

The following existence theorem relates the order of the function to the orders of its Pólya peaks.

THEOREM A. *Let $G(t)$ have order λ and lower order μ $(\mu < \infty$, $\lambda \leqq \infty)$. Then with every finite α such that*

$$\mu \leqq \alpha \leqq \lambda,$$

it is possible to associate a sequence of Pólya peaks of order α.

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