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TIME DECAY AND THE BORN SERIES

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ABSTRACT. A time decay estimate from scattering theory always implies the convergence of the Born series at high energies. That is, if H_0 and V are selfadjoint operators in Hilbert space, and $V\exp(-itH_0)$ is integrable (in a certain sense), then the series expansion of $(H_0 + V - \lambda \pm iO)^{-1}$ in terms of $V(H_0 - \lambda \pm iO)^{-1}$ converges for sufficiently large λ. This abstract result is applied to Schrödinger operators $-\Delta + V$, generalizing work of Zemach and Klein.

1. **Introduction.** Let H_0 be a selfadjoint operator acting in a Hilbert space $\mathfrak{H}$. Consider its resolvent $(H_0 - z)^{-1}$, for z not real. If H_0 has absolutely continuous spectrum, its resolvent will have boundary values $(H_0 - \lambda \pm iO)^{-1}$ for λ real. The values of these operators will lie in a larger space.

In a perturbation problem we consider another selfadjoint operator $H_0 + V$. Then $(H_0 + V - z)^{-1}$ exists for z not real and we may ask about its boundary values. (They occur in expressions for the S operator and wave operators, as well as for spectral representations and spectral projections.) The most elementary approach is through the Born series

$$(H_0 + V - \lambda \pm iO)^{-1} = (H_0 - \lambda \pm iO)^{-1} \sum_{n=0}^{\infty} (-V(H_0 - \lambda \pm iO)^{-1})^n.$$

If this converges for some range of λ, then $H_0 + V$ must have absolutely continuous spectrum there.

One may often expect convergence for all λ when V is sufficiently small. However, this is a very special case, since in general $H_0 + V$ may have eigenvalues in addition to continuous spectrum. On the other hand, it is known that the Born series gives a useful approximation for sufficiently large energies λ. It will be shown here that whenever $V\exp(-itH_0)$ is integrable with respect to t, the Born series converges for sufficiently high energies, whatever the strength of the coupling. The advantage of this criterion is that estimates on the norm of $V\exp(-itH_0)$ for large t are available from time dependent scattering theory. However, the question of measur-

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