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p-SUBGROUPS OF CORE-FREE QUASINORMAL SUBGROUPS

FLETCHER GROSS [1]

1. **Introduction.** The main object of this paper is to obtain bounds on the nilpotence class and derived length of a core-free quasinormal subgroup. Here the subgroup H of G is quasinormal in G if $HK = KH$ for each subgroup K of G; H is core-free if H contains no nonidentity normal subgroup of G. Since Itô and Szép [3] proved that a core-free quasinormal subgroup of a finite group is nilpotent, the problem of determining the class and derived length of the core-free quasinormal subgroup H of the finite group G is equivalent to the problem of determining the class and derived length of the p-subgroups of H. The principal result of the present paper is that if H is a core-free quasinormal subgroup of the (possibly infinite) group G and P is a subgroup of H generated by elements of order dividing p^n where p is a prime, then $x^{p^n} = 1$ for all x in P, P is nilpotent of class $\leqq$ Max $\{1, p^{n-1} - 1\}$, and $d(P)$, the derived length of P, is $\leqq [(n+1)/2]$ if $p = 2$, and $d(P) \leqq n$ if $p > 2$.

Bradway, Gross, and Scott [1] proved that if p is a prime and n is a positive integer $< p$, then there is a finite p-group which contains a core-free quasinormal subgroup of class n and exponent p^2. Thus the upper bound on the class given above is best-possible when $n \leqq 2$. In Theorem 5.2 of this paper it is shown that if p is a prime and n is a positive integer, then there is a finite p-group which contains a core-free quasinormal subgroup of class n and exponent $< np^3$. This theorem not only shows that for any fixed prime p the class of a core-free quasinormal p-subgroup can be arbitrarily large (previously, I do not believe it even was known if a core-free quasinormal 2-subgroup could be nonabelian), but also implies that for $n > 2$ there is a finite p-group which contains a core-free quasinormal subgroup of exponent p^n and class p^{n-2}. Hence if our upper bound on the class is too big, it is too big by less than a factor of p.

2. **Notation and assumed results.** If S is a subset of the group G, then $\langle S \rangle$ is the subgroup of G generated by the elements of S. If H is a

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