ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 1, Number 3, Summer 1971

SPECTRAL REPRESENTATION OF SELFADJOINT DILATIONS OF SYMMETRIC OPERATORS WITH PIECEWISE C^2 SPECTRAL FUNCTIONS

RICHARD C. GILBERT[1]

ABSTRACT. Let A be a simple closed symmetric operator with deficiency index (1, 1) in a Hilbert space $\mathfrak{H}$. Suppose A has a selfadjoint extension A_0 in $\mathfrak{H}$ for which $\rho_0(t) = (E_0(t)g_0, g_0)$ is piecewise C^2, where $E_0(t)$ is the spectral function of A_0, and g_0 is an element in a deficiency subspace of A. Under this assumption, a spectral representation is given for all the selfadjoint extensions and minimal selfadjoint dilations of A. The procedure used is a generalization of that used when A is a Sturm-Liouville operator on $[0, \infty)$ in the limit point case at ∞. The spectral representation clarifies the nature of the spectrum and spectral multiplicity of A^+.

1. Introduction. Let A be a simple closed symmetric operator with deficiency index (1, 1) in the Hilbert space $\mathfrak{H}$. If A^+ is a selfadjoint operator in a Hilbert space $\mathfrak{H}^+$ such that $\mathfrak{H} \subset \mathfrak{H}^+$ and $A \subset A^+$, then A^+ is called a *selfadjoint extension* of A wherever $\mathfrak{H} = \mathfrak{H}^+$, and A^+ is called a *selfadjoint dilation* whenever $\mathfrak{H}$ is properly contained in $\mathfrak{H}^+$. A^+ is called a *minimal* selfadjoint dilation if A^+ is not reduced by any nontrivial subspace of $\mathfrak{H}^+ \ominus \mathfrak{H}$. It is the purpose of this article to present an expansion theorem (Theorem 1) and a spectral representation theorem (Theorem 2) for the selfadjoint extensions and dilations of A. These theorems are analogs of the eigenfunction expansion and spectral representation theorems which can be proved when A is a Sturm-Liouville differential operator on $[0, \infty)$ in the limit point case at ∞. (See, for example, Štraus [7].) In the spectral representation theorem a spectral matrix corresponding

Received by the editors October 22, 1969.

AMS 1969 subject classifications. Primary 4730, 4740; Secondary 4760.
Key words and phrases. Spectral representation, selfadjoint dilations of symmetric operators, selfadjoint extensions of symmetric operators, Sturm-Liouville operators, symmetric operator, Weyl limit point case, eigenfunction expansion, spectrum, spectral multiplicity, spectral matrix, generalized resolvent, spectral function, Stieltjes inversion formula, multiplication operator, fundamental solution.

[1]The author expresses appreciation to California State College, Fullerton, for a research leave during which this work was performed. The work was also supported in part by the National Science Foundation, Grant No. GP 12886.