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PRIMARY COHOMOLOGY OPERATIONS IN *BSJ*

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I. **Introduction.** The study of fiber spaces and fiber bundles has led to several different definitions of equivalence. Two of the most important are "fiber homotopy equivalence" for Hurewicz spherical fiber spaces [8, p. 100] and "bundle equivalence" for spherical fiber bundles [8, p. 92]. If X is a reasonable space, then the set of classes of stable oriented spherical Hurewicz fiber spaces over X is a group, called $\tilde{K}SF(X)$; also the set of classes of stable oriented spherical fiber bundles is a group, called $\tilde{K}SO(X)$.

The contravariant functors $\tilde{K}SF$ and $\tilde{K}SO$ are representable. This means that there are spaces BSF and BSO such that there exist natural isomorphisms $[\ ;BSF] = \tilde{K}SF$ and $[\ ;BSO] = \tilde{K}SO$ when the functors are restricted to a reasonable class of spaces.

For the rest of this introduction, we shall use slightly nonstandard notation. This will serve two purposes. First, it will help to distinguish between the J homomorphism and the contravariant functor which J induces. Second, it will allow our notation to be consistent.

There is a natural transformation $J:\tilde{K}SO \to \tilde{K}SF$, namely the map that associates to each class of bundles over X the class of fiber spaces which includes it. This transformation is the stable J-homomorphism. We use the symbol $\tilde{K}SJ(X)$ to denote the group $J[\tilde{K}SO(X)]$. Thus $\tilde{K}SJ$ is a contravariant functor (usually denoted by J). Adams has shown that $\tilde{K}SJ$ is not a representable functor [1].

Recently there has been some interest in spaces which are "approximately" classifying spaces for the functor $\tilde{K}SJ$. No space does the job perfectly, the argument goes, but some spaces do a better job than others. A good approximation should behave as much as possible as BSJ would behave if it existed. For example, because of the commutative diagram

$$\begin{tikzcd} \tilde{K}SO \arrow[rr, "J"] \arrow[dr, "J"'] & & \tilde{K}SF \\ & \tilde{K}SJ \arrow[ur] & \end{tikzcd}$$

there should exist a corresponding homotopy commutative diagram

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