

SUFFICIENT CONDITIONS FOR A GROUP TO BE A DIRECT SUM OF CYCLIC GROUPS¹

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1. Introduction. This paper, except possibly the last section, may be regarded as expository. Although substantial extensions of the known results are obtained herein, these new results are more or less immediate corollaries of the author's uniqueness theorem for totally projective groups [5]. However, the direct proofs given here are much simpler than the proof of the uniqueness theorem.

Sufficient conditions on a commutative group in order that it be a direct sum of cyclic groups have been sought for a long time. Although several such conditions have been found, better and more refined conditions are still desired. The oldest and probably the best known condition that implies that a commutative group is a direct sum of cyclic groups is that the group be finite. In searching for conditions that make an infinite commutative group a direct sum of cyclic groups, one may, of course, immediately restrict the problem to the primary and torsion-free cases. We concentrate in this paper on the primary case, that is, on finding sufficient conditions for a primary group to be a direct sum of cyclic groups. However, we remark that in the torsion-free case, as well as the primary case, there are some deep and important problems in abelian groups that amount basically to the question of whether or not a certain condition is sufficient for a group to be a direct sum of cyclic groups. Consider, for example, the famous problems of Baer and Whitehead; two recent papers of interest on these problems are [2] and [3].

An element g in an additively written p -primary group G is said to be divisible by p^n if there is a solution in G to the equation $p^n x = g$. The element g has *height* n if g is divisible by p^n but not by p^{n+1} . If g is divisible by p^n for each positive integer n , then we say that g has infinite height. Naturally, we say that G is without elements of infinite height if zero is the only element that has infinite height. Observe that in order for G to be a direct sum of cyclic groups it is necessary that G be without elements of infinite height. The following result was a major contribution to the theory of infinite commutative groups in its early development.

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