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A QUADRATURE METHOD FOR SYSTEMS OF CAUCHY SINGULAR INTEGRAL EQUATIONS

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Dedicated to Professor Giuseppe Mastroianni for his 70th birthday

ABSTRACT. The aim of this paper is to propose a numerical method approximating the solutions of a system of CSIE. The stability and the convergence of the method are proved in weighted L^2 spaces. An application to the numerical resolution of CSIE on curves is also given. Finally, some numerical tests confirming the error estimates are shown.

1. Introduction. Systems of singular integral equations with Cauchy type kernels may be found in the formulation of many boundary value problems. In many known physical problems of practical interest, the coefficients of the equations are constant. The general theory of such systems is given in [11, 14] (see also the references therein).

In this paper we are interested in the numerical solution of systems of the following type

(1.1)
$$a_j F_j(\tau) + \frac{b_j}{\pi} \int_{-1}^{1} \frac{F_j(t)}{t - \tau} dt$$

 $+ \sum_{k=1}^{n} \int_{-1}^{1} h_{jk}(\tau, t) F_k(t) dt = G_j(\tau), \quad |\tau| < 1,$
 $j = 1, \dots, n,$

where h_{jk} and G_j , j, k = 1, ..., n, are known complex-valued functions defined on $[-1, 1]^2$ and [-1, 1], respectively, and F_j , j = 1, ..., n, are the unknowns.

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