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PROPERTIES OF THE SINGLE LAYER POTENTIAL FOR THE TIME FRACTIONAL DIFFUSION EQUATION

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ABSTRACT. This paper investigates the basic properties of the single layer potential for the time fractional diffusion equation (TFDE) in a bounded domain with Lyapunov boundary. We prove the continuity of the single layer potential across the boundary, that the normal derivative of the single layer potential satisfies the usual jump relation known for the heat equation and that the single layer potential is Hölder continuous. These properties are essential when investigating the boundary integral equation corresponding to TFDE.

Although the results are well known for the potentials of the heat equation corresponding to the case $\alpha = 1$, the proofs of the same properties seem not to be available in the case $0 < \alpha < 1$. Also, even if the proofs follow the same lines as in the case $\alpha = 1$, there are some additional difficulties to overcome. First of all, there is no explicit formula for the fundamental solution in terms of elementary functions unlike in the case of the heat potential. Secondly, the behavior of the Fox H-functions is different for small and large arguments. However, the known asymptotic behavior of the Fox H-functions allows us to prove the above-mentioned properties.

1. Introduction. We are interested in the boundary integral solution of the time fractional diffusion equation

$$
\begin{aligned}
\partial_t^\alpha \Phi - \Delta_x \Phi = 0 \qquad & \text{in } Q_T = \Omega \times (0,T), \\
\Phi = g, \qquad & \text{on } \Sigma_T = \Gamma \times (0,T), \\
\Phi(x,0) = 0, \quad & x \in \Omega,
\end{aligned} \tag{1}
$$

with the single layer potential. In problem (1) the domain $\Omega \subset \mathbf{R}^n$ is assumed to be bounded with the boundary $\Gamma \in \mathcal{C}^{1+\lambda}$, $0 < \lambda < 1$, and

$$
\partial_t^\alpha u(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} u'(\tau)\, \mathrm{d}\tau \tag{2}
$$

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