

STOCHASTIC VOLTERRA EQUATIONS IN WEIGHTED SPACES

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ABSTRACT. In the following paper, we provide a stochastic analogue to work of Shea and Wainger by showing that when the measure and state-independent diffusion coefficient of a linear Itô-Volterra equation are in appropriate L^p -weighted spaces, the solution lies in a weighted L^p -space in both an almost sure and moment sense.

1. Introduction. This paper examines the asymptotic stability and decay rates, in various modes of stochastic convergence, of solutions of stochastically perturbed Volterra equations to the equilibrium solution of a related unperturbed deterministic Volterra equation. For deterministic equations the phenomenon of asymptotic stability has been shown to be distinct from that of exponential stability. These phenomena were shown to coincide in linear Volterra integrodifferential equations by Murakami [30, 31] if and only if the kernel lies in an exponentially weighted L^1 -space. On the other hand, non-exponential rates of decay in spaces of integrable functions with general weights have been considered by Gelfand et al. [12], Shea and Wainger [32] and Jordan and Wheeler [19], with extensions of the last paper presented in Jordan, Staffans and Wheeler [18]. An account of this research is summarized in Gripenberg et al. [13].

Asymptotic stability results for stochastic functional and evolution equations without concern for the rate of decay appear in, e.g., Hauss-

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