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ON THE DUALITY OF THE POTENTIAL METHOD AND THE POINT SOURCE METHOD IN INVERSE SCATTERING PROBLEMS

JIJUN LIU AND ROLAND POTTHAST

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This paper is dedicated to Prof. Dr. Rainer Kress
on the occasion of his 65th birthday.

ABSTRACT. The reconstruction of scattered wave from its far-field pattern is of great importance in inverse scattering problems. The classic *potential method* due to Kirsch and Kress is a well known scheme by solving an integral equation of the first kind with respect to a density function, which relates the scattered wave to its far-field pattern. In recent years, a filtering scheme known as *point source method*, is also well developed, which is based on the point source decomposition and the reciprocity principle. This paper aims to consider the *quantitative relation* between these two regularizing methods. We prove that these two schemes will generate *exactly the same approximate solution* when used with identical geometric setup and if their own regularizing parameters are taken as a constant multiple (a *golden rule*). Our key step is to employ an *adjoint relation* between the Herglotz wave operator and the far-field operator. Further we provide estimates of the solutions with regularization parameters different from the golden rule. As illustration and for practical testing of these results numerical examples are presented to show the numerical equivalence of these two methods.

1. Introduction. Inverse problems for acoustic and electromagnetic waves play an important role in many scientific and engineering applications. One of the important topics in this area is the reconstruction of scattered wave outside of the scatterer D from its far-field pattern. The well known Rellich lemma guarantees the unique deter-

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