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CONDITION NUMBER ESTIMATES FOR COMBINED POTENTIAL BOUNDARY INTEGRAL OPERATORS IN ACOUSTIC SCATTERING

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Dedicated to Rainer Kress on the occasion of his 65th birthday.

ABSTRACT. We study the classical combined field integral equation formulations for time-harmonic acoustic scattering by a sound soft bounded obstacle, namely the indirect formulation due to Brakhage-Werner/Leis/Panič, and the direct formulation associated with the names of Burton and Miller. We obtain lower and upper bounds on the condition numbers for these formulations, emphasising dependence on the frequency, the geometry of the scatterer, and the coupling parameter. Of independent interest we also obtain upper and lower bounds on the norms of two oscillatory integral operators, namely the classical acoustic single- and double-layer potential operators.

1. Introduction. In this paper we consider the classical problem of scattering of a time-harmonic acoustic wave by a bounded, sound soft obstacle occupying a compact set $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3) with Lipschitz boundary Γ. The wave propagates in the exterior domain $\Omega_e = \mathbb{R}^d \setminus \Omega$ and we suppose that the medium of propagation in Ω_e is homogeneous, isotropic and at rest, and that a time harmonic ($\mathrm{e}^{-\mathrm{i}\omega t}$ time dependence) pressure field u^i is incident on Ω. Denoting by $c > 0$ the speed of sound, we assume that u^i is an entire solution of the Helmholtz (or reduced wave) equation with *wave number* $k = \omega/c > 0$.

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