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THE LINEAR SAMPLING METHOD REVISITED

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This paper is dedicated to Professor Rainer Kress on the occasion of his 65th birthday.

ABSTRACT. This paper is concerned with convergence results for the Linear Sampling method, a method in inverse scattering theory characterizing an unknown obstacle directly through an indicator function computed from the data. Three seperate but related results are shown. Firstly, sufficient conditions are formulated for the choice of the regularization parameter that guarantee that the method converges in the presence of noise for a sampling point inside the obstacle. Secondly, a new, very strong connection to the related Factorization method is proved. Thirdly, for the first time the behaviour of the indicator function for sampling points outside the obstacle is adequately explained.

1. Introduction. Inverse scattering problems for time-harmonic acoustic waves have been a popular research subject for a long time [**7**]. Among the methods employed for their solution are iterative Newton-type methods, where differentiability with respect to the scatterer is exploited, decomposition methods separating the reconstruction of the scattered fields from the reconstruction of an obstacle or inhomogeneity, and sampling methods. Such methods allow the computation of an indicator function characterizing the unknown obstacle directly from the data. It is the latter class of methods we are concerned with in this paper.

Theory on sampling methods started with the *Linear Sampling method*, first introduced in [**5, 6**]. It requires the knowledge of the far field pattern $u^\infty(\hat{x}, d)$ of the scattered field for incident plane waves of all possible directions $d \in \mathbb{S}^2$ and all directions of observation $\hat{x} \in \mathbb{S}^2$. Here, $\mathbb{S}^2$ denotes the unit sphere in $\mathbb{R}^3$. From this data, the so-called

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