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ON THE CONNECTION BETWEEN MOLCHAN-GOLOSOV AND MANDELBROT-VAN NESS REPRESENTATIONS OF FRACTIONAL BROWNIAN MOTION

CÉLINE JOST

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ABSTRACT. We prove analytically a connection between the generalized Molchan-Golosov integral transform, see [**4**, Theorem 5.1], and the generalized Mandelbrot-Van Ness integral transform, see [**8**, Theorem 1.1], of fractional Brownian motion (fBm). The former changes fBm of arbitrary Hurst index K into fBm of index H by integrating over $[0,t]$, whereas the latter requires integration over $(-\infty,t]$ for $t>0$. This completes an argument in [**4**], where the connection is mentioned without full proof.

1. Introduction. The *fractional Brownian motion with Hurst index* $H \in (0,1)$, or H-fBm, is the continuous, centered Gaussian process $(B_t^H)_{t\in\mathbf{R}}$ with $B_0^H=0$, almost surely, and

$$\mathrm{Cov}_{\mathbf{P}}\left(B_s^H, B_t^H\right) = \frac{1}{2}\left(|s|^{2H} + |t|^{2H} - |t-s|^{2H}\right), \quad s,t \in \mathbf{R}.$$

H-fBm is H-self-similar and has stationary increments. For $H=1/2$, fractional Brownian motion is standard Brownian motion and denoted by W. FBm is interesting from a theoretical point of view, since it is fairly simple, but neither a Markov process, nor a semi-martingale. Recently, the process has been studied extensively in connection to various applications, for example in finance and telecommunications. Important tools when working with fBm are its integral representations:

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