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ON INTEGRAL EQUATIONS OF THE FIRST KIND WITH LOGARITHMIC KERNELS

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ABSTRACT. The existence-uniqueness of the solution and its behaviour for one-dimensional integral equations of the first kind with logarithmic kernels are investigated. The analysis is based on the transfinite diameter or logarithmic capacity and spaces deriving from Fourier series. The uniqueness results apply to any closed bounded subset of the plane. The other results apply to open arcs, polygons and other regions with piecewise-smooth boundaries.

0. Introduction. In this paper, the integral equation of the first kind,

$$(1) \qquad -\int_\Gamma \log|x-y|g(y)\,dy = f(x), \quad x\in\Gamma,$$

will be investigated, where dy denotes the arc-length element at $y\in\Gamma$. Γ is a curve in the plane, smooth or piecewise smooth, open or closed, such as an interval, circle or polygon [**5**]. Some discussion will also be given for the more general case, appearing in [**24**], [**2**], [**38**],

$$(1^*) \qquad -\int_\Gamma [\log|x-y| + F(x,y)]\,g(y)\,dy = f(x), \quad x\in\Gamma,$$

where $F(x,y)$ is a function of $x, y\in\Gamma$ which is smoother than $\log|x-y|$. We will give existence-uniqueness theorems for the solutions and show their smoothness and singular character.

The integral equation of the first kind in the form (1) follows from the representation of a harmonic function by single-layer potentials, or by the direct boundary integral equation method (BIEM) for plane Dirichlet boundary value problems, both of great importance in engineering [**29**, **4**, **18**, **5**, **6**, **7**]. The classical boundary integral equations usually appeared in the form of second kind integral equations [**34**, **29**],because of the simplicity of Fredholm theory. But in the last decade engineers and mathematicians have noticed that boundary integral equations of