

## BASS NUMBERS OVER LOCAL RINGS VIA STABLE COHOMOLOGY

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To Jürgen Herzog on his 70th birthday.

**ABSTRACT.** For any non-zero finite module  $M$  of finite projective dimension over a Noetherian local ring  $R$  with maximal ideal  $\mathfrak{m}$  and residue field  $k$ , it is proved that the natural map  $\mathrm{Ext}_R(k, M) \rightarrow \mathrm{Ext}_R(k, M/\mathfrak{m}M)$  is non-zero when  $R$  is regular and is zero otherwise. A noteworthy aspect of the proof is the use of stable cohomology. Applications include computations of Bass series over certain local rings.

**1. Introduction.** Let  $(R, \mathfrak{m}, k)$  denote a commutative Noetherian local ring with maximal ideal  $\mathfrak{m}$  and residue field  $k$ ; when  $R$  is not regular we say that it is *singular*.

This article revolves around the following result:

**Theorem.** *If  $(R, \mathfrak{m}, k)$  is a singular local ring and  $M$  an  $R$ -module of finite projective dimension, then  $\mathrm{Ext}_R(k, \pi^M) = 0$  for the canonical map  $\pi^M : M \rightarrow M/\mathfrak{m}M$ .*

Special cases, known for a long time, are surveyed at the end of Section 2. Even in those cases our proof is new. It utilizes a result of Martsinkovsky [11] through properties of Vogel's stable cohomology functors [3, 6] recalled in Section 1. It also suggests extensions to DG modules over certain commutative DG algebras; these will be discussed in [2]. Applications of the theorem include new criteria for regularity of local rings (in Section 2) and explicit computations of Bass numbers of modules (in Section 3).

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