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NEW FREE DIVISORS FROM OLD

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To Jürgen Herzog on his 70th birthday.

ABSTRACT. We present several methods to construct or identify families of free divisors such as those annihilated by many Euler vector fields, including binomial free divisors, or divisors with triangular discriminant matrix. We show how to create families of quasihomogeneous free divisors through the chain rule or by extending them into the tangent bundle. We also discuss whether general divisors can be extended to free ones by adding components and show that adding a normal crossing divisor to a smooth one will not succeed.

1. Introduction. The goal of this note is to describe some basic operations that allow to construct new free divisors from given ones, and to classify toric free surfaces and binomial free divisors. We mainly deal with weighted homogeneous polynomials over a field of characteristic 0, although several statements and constructions generalize to power series.

A (formal) *free divisor* is a reduced polynomial (or power series) f in variables $x_1, \dots, x_n$ over a field K such that its Jacobian ideal $J(f) = (\partial f/\partial x_1), \dots, (\partial f/\partial x_n) + (f)$ is perfect of codimension 2 in the polynomial or power series ring. For generalities about free divisors and their importance in singularity theory, we refer to, say, [**2**] and the references therein.

A determinantal characterization of free divisors is due to Saito [**10**]: a reduced polynomial f is a free divisor if and only if there exists a matrix A of size $n \times n$ with entries in the relevant polynomial or power series ring such that $\det(A) = f$ and $(\nabla f)A \equiv 0 \bmod (f)$, where

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