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TRIPLETS OF PURE FREE SQUAREFREE COMPLEXES

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Dedicated to Jürgen Herzog on the occasion of his 70th birthday.

ABSTRACT. On the category of bounded complexes of finitely generated free squarefree modules over the polynomial ring S, there is the standard duality functor $\mathbf{D} = \mathrm{Hom}_S(-, \omega_S)$ and the Alexander duality functor $\mathbf{A}$. The composition $\mathbf{A} \circ \mathbf{D}$ is an endofunctor on this category, of order three up to translation. We consider complexes $F_\bullet$ of free squarefree modules such that both $F_\bullet, \mathbf{A} \circ \mathbf{D}(F_\bullet)$ and $(\mathbf{A} \circ \mathbf{D})^2(F_\bullet)$ are pure, when considered as singly graded complexes. We conjecture: i) the existence of such triplets of complexes for given triplets of degree sequences, and ii) the uniqueness of their Betti numbers, up to scalar multiple. We show that this uniqueness follows from the existence, and we construct such triplets if two of its degree sequences are linear.

Introduction. *Pure free resolutions* are free resolutions over the polynomial ring S of the form

$$S(-d_0)^{\beta_0} \leftarrow S(-d_1)^{\beta_1} \leftarrow \cdots \leftarrow S(-d_r)^{\beta_r}.$$

Their Betti diagrams have proven to be of fundamental importance in the study of Betti diagrams of graded modules over the polynomial ring. Their significance was put to light by the Boij-Söderberg conjectures, [**2**]. The existence of pure resolutions were first proved by Eisenbud, the author and Weyman in [**7**] in characteristic zero, and by Eisenbud and Schreyer in all characteristics, [**8**]. Later, the methods of [**8**] were made more explicit and put into a larger framework, called tensor complexes, by Berkesch et al. [**1**].

The Boij-Söderberg conjectures, settled in full generality in [**8**], concerns the stability theory of *Betti diagrams* of graded modules, i.e.,

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