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ON THE EQUALITY OF ORDINARY AND SYMBOLIC POWERS OF IDEALS

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ABSTRACT. We consider the following question concerning the equality of ordinary and symbolic powers of ideals. In a regular local ring, if the ordinary and symbolic powers of a prime ideal are the same up to its height, then are they the same for all powers? We provide supporting evidence of a positive answer for classes of prime ideals defining monomial curves or rings of low multiplicities.

1. Introduction. Let R be a Noetherian local ring of dimension d, and let P be a prime ideal of R. For a positive integer n, the nth symbolic power of P, denoted by $P^{(n)}$, is defined as

$$P^{(n)} := P^n R_P \cap R = \{x \in R \mid \text{there exists an } s \in R \setminus P,\ sx \in P^n\}.$$

One readily sees from the definition that $P^n \subseteq P^{(n)}$ for all n, but they may not be equal in general. Comparing the ordinary and symbolic powers of ideals is a subject of interest in both commutative algebra and algebraic geometry, see for instance [**2, 8, 10–13, 15, 21**]. In this paper, we are interested in criteria for the equality. In particular, we would like to know if $P^n = P^{(n)}$ for all n up to some value implies that they are equal for all n. The following question was posed by Huneke in this regard.

Question 1.1. Let R be a regular local ring of dimension d, and let P be a prime ideal of height $d-1$. If $P^n = P^{(n)}$ for all $n \leq d-1$, then is $P^n = P^{(n)}$ for all n?

An affirmative answer to Question 1.1 is equivalent to P being generated by a regular sequence [**7**]. Furthermore, it is equivalent to showing that if $P^n = P^{(n)}$ for all $n \leq d-1$, then the analytic spread of P is $d-1$. This is not known even for the defining ideals of monomial

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