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ZERO DIVISOR GRAPHS FOR MODULES OVER COMMUTATIVE RINGS

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ABSTRACT. In this article, we give several generalizations of the concept of zero-divisor elements in a commutative ring with identity to modules. Then, for each R-module M, we associate three undirected (simple) graphs $\Gamma^*({}_RM) \subseteq \Gamma({}_RM) \subseteq \Gamma_*({}_RM)$ which, for $M = R$, all coincide with the zero-divisor graph of R. The main objective of this paper is to study the interplay of module-theoretic properties of M with graph-theoretic properties of these graphs.

0. Introduction. Let R be a commutative ring with identity and $Z(R)$ its set of zero divisors. In [**6**], Anderson and Livingston associated an undirected (simple) graph $\Gamma(R)$ to R with vertices $Z(R)^* := Z(R) \setminus \{0\}$ and with two distinct vertices x and y adjacent if $xy = 0$, and then studied the relationship between the properties of $\Gamma(R)$ and R. This graph is defined somewhat differently from the graph introduced by Beck [**8**], who took the set of vertices to be all of R. Recently, this subject has received a good deal of attention from several authors assigning a graph to a ring or a group and then studying the algebraic properties of these objects via their associated graphs; see, for instance, [**1–8, 13, 14, 18, 19, 23**]. Moreover, Redmond in [**20**] has considered the zero-divisor graph for arbitrary rings (see also [**1**]). In the present article, we introduce and study several generalizations of zero-divisor graphs to modules M over a commutative ring R which, for $M = R$, all coincide with $\Gamma(R)$ (the zero-divisor graph of R). Our main objective is to establish connections between module theoretic properties with the properties of associated graphs.

Throughout, all rings are commutative with identity elements, and all modules are unitary. The symbol $\subseteq$ denotes containment, and $\subset$ denotes proper containment for sets. If N is a submodule (respectively,

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