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PRÜFER DOMAINS WITH CLIFFORD CLASS SEMIGROUP

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ABSTRACT. Bazzoni's conjecture states that the Prüfer domain R has finite character if and only if R has the property that an ideal of R is finitely generated if and only if it is locally principal. In [**5**] the authors use the language and results from the theory of lattice-ordered groups to show that the conjecture is true. In this article we supply a purely ring theoretic proof.

1. Bazzoni's conjecture. Throughout all integral domains are assumed to be commutative.

For an integral domain R, $\mathscr{F}(R)$ denotes the semigroup of fractional ideals of R (under ideal multiplication) while $\mathscr{P}(R)$ denotes the subsemigroup consisting of principal ideals. The class semigroup of R is the factor semigroup $\mathscr{F}(R)/\mathscr{P}(R)$ and is denoted $\mathscr{S}(R)$. A semigroup S is called a *Clifford semigroup* when every element is regular in the sense of von Neumann, that is, for every $a \in S$ there is an $s \in S$ for which $a^2 s = a$. The domain R is called a *Clifford regular domain* when $\mathscr{S}(R)$ is Clifford regular.

In the article [**1**] Bazzoni proved that if a Prüfer domain has finite character (that is, every nonzero element belongs to a finite number of maximal ideals), then $\mathscr{S}(R)$ is a Clifford semigroup, and in turn, if $\mathscr{S}(R)$ is a Clifford semigroup, then R satisfies $(*)$ (defined below). In a later article, [**2**], she was able to show that if $\mathscr{S}(R)$ is a Clifford semigroup, then R has finite character. In [**1**] and then again in [**2**] she proposed the following:

Conjecture. *A Prüfer domain satisfies property $(*)$ if and only if R has finite character.*

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