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A CHARACTERIZATION OF COFINITE COMPLEXES OVER COMPLETE GORENSTEIN DOMAINS

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ABSTRACT. Let R be a complete Gorenstein local domain, J an ideal of R of dimension one, and $N^\bullet$ a complex of R-modules bounded below. In this paper, we prove that $N^\bullet$ is a J-cofinite complex if and only if $H^i(N^\bullet)$ is a J-cofinite module for all i. Consequently, this assertion affirmatively answers the fourth question in [**4**, page 149] for an ideal of dimension one over a complete Gorenstein local domain.

1. Introduction. We assume that all rings are commutative and Noetherian with identity throughout this paper.

In this paper, we shall prove the following theorem.

Theorem 1. *Let R be a complete Gorenstein local domain of dimension d, and let J be an ideal of R of dimension one. Let $N^\bullet$ be a complex of R-modules in $\mathcal{D}^+(R)$, where $\mathcal{D}^+(R)$ is the derived category consisting of complexes bounded below. Then $N^\bullet$ is J-cofinite if and only if $H^i(N^\bullet)$ is in $\mathcal{M}(R,J)_{\rm cof}$ for all i, where $\mathcal{M}(R,J)_{\rm cof}$ is a category of J-cofinite modules (see Definition 3 below).*

The following question is proposed in the paper [**4**, Section 2]:

Question 1. Let R be a regular ring of dimension d and J an ideal of R. Suppose that R is complete with respect to the J-adic topology. Then does there exist an abelian category $\mathcal{M}_{\rm cof}$ consisting of R-modules, such that elements $N^\bullet \in \mathcal{D}(R,J)_{\rm cof}$ are characterized by the property "$H^i(N^\bullet) \in \mathcal{M}_{\rm cof}$" for all i? Here we denote $\mathcal{D}(R,J)_{\rm cof}$ is the essential image of $\mathcal{D}_{ft}(R)$ by the J-dualizing functor (see Definition 1 below for the definition of the dualizing functor).

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