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ON THE SECOND POWERS OF STANLEY-REISNER IDEALS

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ABSTRACT. In this paper, we study several properties of the second power I_Δ^2 of a Stanley-Reisner ideal I_Δ of any dimension. As the main result, we prove that S/I_Δ is Gorenstein whenever S/I_Δ^2 is Cohen-Macaulay over any field K. Moreover, we give a criterion for the second symbolic power of I_Δ to satisfy (S_2) and to coincide with the ordinary power, respectively. Finally, we provide new examples of Stanley-Reisner ideals whose second powers are Cohen-Macaulay.

0. Introduction. It is proved in [**24**] that a simplicial complex Δ is a complete intersection if the third power I_Δ^3 of its Stanley-Reisner ideal is Cohen-Macaulay, using a result in [**17, 27**]. On the other hand, there is a simplicial complex Δ which is not a complete intersection such that I_Δ^2 is Cohen-Macaulay. The simplicial complex associated with a pentagon is such an example. Among one-dimensional simplicial complexes, the above example is a unique one, as shown in [**16**]. As for the two-dimensional case, such simplicial complexes are classified in [**26**]. In [**17**] a characterization of Cohen-Macaulayness of the second symbolic power $I_\Delta^{(2)}$ is given.

A main motivation of this paper is to study the Cohen-Macaulayness of the second ordinary powers of Stanley-Reisner ideals of any dimension. We consider the following two questions:

(1) What constraints does Cohen-Macaulayness of I_Δ^2 impose upon a simplicial complex Δ?

(2) Do there exist *many* simplicial complexes Δ such that I_Δ^2 are Cohen-Macaulay?

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