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DISCRIMINANT COAMOEBAS IN DIMENSION TWO

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ABSTRACT. This paper deals with coamoebas, that is, images of algebraic varieties under coordinatewise argument mappings. We focus on the case of A-discriminant varieties where $A \subset \mathbf{Z}^n$ has cardinality $n+3$ and satisfies a natural genericity hypothesis. We give a very explicit description of the coamoeba as the union of two mirror images of a (possibly non-convex) polygon, easily constructed from the Gale transform of A. We also give an area formula for the coamoeba, showing that the coamoeba is intimately related to a certain zonotope. In particular, we show how the coamoeba and this zonotope together yield a covering of the torus with multiplicity equal to the volume of the convex hull of A.

1. Introduction. The objects that we study in this article are special algebraic curves defined by reduced, or inhomogeneous, A-discriminantal polynomials; and more specifically, the coamoebas of these A-discriminantal curves. Such a coamoeba is the image of the curve under the simple mapping that takes each complex coordinate to its argument. The notion of A-discriminants was originally introduced by Gelfand, Kapranov, and Zelevinsky, and the monograph [**3**] contains a thorough account of the subject. Here we just recall a few facts that will help put our results in proper perspective. The starting point is a finite configuration A of points $\alpha_1, \alpha_2, \dots, \alpha_N$ in $\mathbf{Z}^n$, and the corresponding family of Laurent polynomials

$$f(x) = \sum_{k=1}^{N} a_k\, x^{\alpha_k} \in \mathbf{C}[x_1^{\pm 1}, \dots, x_n^{\pm 1}]$$

with exponent vectors from A. Granted some generically satisfied conditions, see [**3**], the set of coefficients vectors $(a_1, \dots, a_N)$, for

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