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DENSE PERIODICITY ON GRAPHS

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ABSTRACT. We establish a Barge-Martin type theorem for graph self-maps for which the set of periodic points is dense.

1. Introduction. The purpose of this paper is to describe graph self-maps for which the set of periodic points is dense. Barge and Martin [**2**] established a structure theorem for maps on the interval with dense periodic points; that is, the twice iterate of such a map is topologically mixing on some countable subintervals and is identical on the other. A similar theorem was proved for tree maps in [**7**].

In this paper, we extend the above to graph self-maps, see Section 3. A motivation for studying graph maps is that higher-dimensional dynamics can often be reduced to a one-dimensional dynamics: this is the case in the study of the structure of attractors of a diffeomorphism, the quotient maps generated by maps on manifolds with an invariant foliation of codimension one and the dynamics of pseudo-Anosov homeomorphisms on a surface.

Throughout this paper, by a *graph*, we mean a *connected* compact one-dimensional polyhedron, and a *tree* is a graph which contains no loops. For a graph G, we denote the sets of endpoints and of branch points of G by $\mathrm{E}\,(G)$ and $\mathrm{B}\,(G)$, respectively. A *map* f is a continuous function; f^0 is the identity map, and for every $n \geq 0$, $f^{n+1} = f^n \circ f$. We denote by $\mathrm{Fix}\,(f)$ and $\mathrm{Per}\,(f)$ the sets of fixed points and of periodic points of f, respectively. A subset K of X is invariant under $f : X \to X$ if $f(K) \subseteq K$, $\mathrm{Int}\, K$ and $\mathrm{Cl}\, K$ denote the interior and closure of K in X, and the orbit of $x \in X$ under f is $\mathrm{Orb}\,_f(x) = \{f^n(x) \mid n \geq 0\}$.

For a natural number S, N_S denotes the least common multiple of the positive integers less than or equal to S.

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