ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 37, Number 6, 2007

WEIERSTRASS' THEOREM IN WEIGHTED SOBOLEV SPACES WITH K DERIVATIVES

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ABSTRACT. We characterize the set of functions which can be approximated by smooth functions and by polynomials with the norm

$$\|f\|_{W^{k,\infty}(w)} := \sum_{j=0}^{k} \left\|f^{(j)}\right\|_{L^\infty(w_j)},$$

for a wide range of (even nonbounded) weights w_j's. We allow a great deal of independence among the weights w_j's.

1. Introduction. If I is any compact interval, Weierstrass's theorem says that $C(I)$ is the largest set of functions which can be approximated by polynomials in the norm $L^\infty(I)$, if we identify, as usual, functions which are equal almost everywhere. There are many generalizations of this theorem, see e.g., the monographs [**20, 23** and the references therein].

In [**24, 28**] we study the same problem with the norm $L^\infty(w)$ defined by

$$\|f\|_{L^\infty(w)} := \text{ess sup}_{x\in\mathbf{R}} |f(x)|w(x), \tag{1}$$

where w is a weight, i.e., a nonnegative measurable function, and we use the convention $0 \cdot \infty = 0$. In [**24**] we improve the theorems in [**28**], obtaining sharp results for a large class of weights, see Theorem 2.1 below. Notice that (1) is not the usual definition of the L^∞ norm

AMS *Mathematics subject classification.* Primary 41A10, 46E35, 46G10.
Keywords and phrases. Weierstrass's theorem, weight, Sobolev spaces, weighted Sobolev spaces.
Research of the first, third and fourth authors partially supported by grants from (MTM 2006-13000-C03-02, MTM 2006-11976 and MTM 2006-26627-E), Spain.
Received by the editors on November 19, 2004, and in revised form on August 5, 2005.