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THE ASYMPTOTIC BEHAVIOR OF NONLINEAR EIGENVALUES

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ABSTRACT. In this paper we study the asymptotic behavior of eigenvalues of the weighted one dimensional p Laplace operator, by using the Prufer transformation. We found the order of growth of the kth eigenvalue, improving the remainder estimate for regular weights.

1. Introduction. In this paper we study the nonlinear eigenvalue problem:

$$-(|u'(x)|^{p-2}u'(x))' = \lambda r(x)|u(x)|^{p-2}u(x), \tag{1.1}$$

in $[0,1]$, with Dirichlet or Neumann boundary conditions. Here, the weight r is a real-valued, positive continuous function, λ is a real parameter, and $1 < p < +\infty$. The spectrum consists on a countable sequence of nonnegative simple eigenvalues $\lambda_1 < \lambda_2 < \cdots < \lambda_k < \cdots$ tending to $+\infty$, see [**5**]. With the same ideas as in [**1**], it was proved in [**4**] that the sequence $\{\lambda_k\}_k$ coincides with the eigenvalues obtained by the Ljusternik Schnirelmann theory.

We define the spectral counting function $N(\lambda)$ as the number of eigenvalues of problem (1.1) less than a given λ:

$$N(\lambda) = \#\{k : \lambda_k \le \lambda\}.$$

The problem of estimating the spectral counting function has a long history in the linear case $p = 2$. See, for instance, [**7, 8**] and the references therein. For $p \neq 2$, the asymptotic behavior of $N(\lambda)$ was obtained in [**4**], by using variational arguments, including a suitable extension of the 'Dirichlet-Neumann bracketing' method. In that

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