

## EXISTENCE OF MONOTONE SOLUTIONS FOR A NONLINEAR QUADRATIC INTEGRAL EQUATION OF VOLTERRA TYPE

ZEQING LIU AND SHIN MIN KANG

**ABSTRACT.** In this paper a sufficient condition for the existence of monotone solutions of the following nonlinear quadratic integral equation of Volterra type

$$x(t) = a(t) + g(x(t)) \int_0^t v(t, s, x(s)) ds, \quad \text{for all } t \in [0, T],$$

is established. Our approach is based on Darbo's fixed point theorem and the measure of noncompactness introduced by Banaś and Olszowy. As applications, some examples to demonstrate our result are given.

**1. Introduction and preliminaries.** We are interested in the existence of monotone solutions for the following nonlinear quadratic integral equation of Volterra type:

$$(1.1) \quad x(t) = a(t) + g(x(t)) \int_0^t v(t, s, x(s)) ds, \quad \text{for all } t \in [0, T],$$

where the functions  $a = a(t)$ ,  $g = g(x)$  and  $v = v(t, s, x)$  appearing in (1.1) are given while  $x = x(t)$  is an unknown function.

It is known that the theory of integral equations has various applications in engineering, mathematical physics, economics and biology. For details, we refer to [1, 2, 10, 16, 17] and the references therein. Within the past 20 year or so, many authors studied the existence of solutions for several classes of nonlinear quadratic integral equations,

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