

K -THEORY AND K -HOMOLOGY OF C^* -ALGEBRAS FOR ROW-FINITE GRAPHS

INHYEOP YI

ABSTRACT. We compute the K -groups and K -homology groups of C^* -algebras of row-finite graphs using the universal covering trees of graphs and Pimsner's six-term exact sequences for KK -groups of crossed products by groups acting on trees.

1. Introduction. Since Cuntz and Krieger introduced a class of C^* -algebras related to subshifts of finite type ([1]), these algebras have been generalized in many ways: C^* -algebras of Smale spaces ([8, 9, 18–20, 23, 26]) from the viewpoint of hyperbolic dynamics, C^* -algebras of row-finite graphs ([11–15, 21]) and C^* -algebras of countably infinite graphs ([5, 6, 21]) from the viewpoint of graph representations of subshifts of finite type, and C^* -algebras of continuous graphs and Cuntz-Pimsner algebras ([2, 10, 17]) from the viewpoint of Hilbert-bimodules. Many of these algebras with appropriate conditions are contained in the bootstrap category of Rosenberg and Schochet [22] so that it is possible to classify these algebras by computing K -groups.

In this paper, we compute the K -groups and K -homology groups of row-finite graph C^* -algebras. Firstly, we remark that Pask, Raeburn and Szymański [14, 21] computed the K -groups of C^* -algebras of row-finite graphs using the canonical gauge action of S^1 , Pimsner-Voiculescu six-term exact sequence for crossed products by \mathbf{Z} and Takai duality. And Drinen and Tomforde [4, 25] computed Ext-groups of C^* -algebras of row-finite graphs with no sinks extending Cuntz and Krieger's method.

We approach the computations of K -groups and K -homology groups from a different direction. The origin of this paper is the author's attempt to understand the works of Kumjian and Pask [11] and

AMS *Mathematics subject classification.* Primary 46L80, 20E08, 19Kxx, Secondary 05C25.

Keywords and phrases. Graph, universal covering tree, fundamental transversal, K -groups, K -homology.

Received by the editors on May 23, 2004, and in revised form on June 2, 2005.